The Unified Model

Let Ω be the set of what can happen in the world, be the allowed trading times, a collection of increasingly fine partitions of Ω representing the information available at each time, and a positive measure with mass 1 on the algebra generated by the partitions.

The *standard model* specifies *prices* , and *cash flows* , where are the available market instruments. Instrument prices are assumed to be perfectly liquid: they can be bought and sold at the same price in any amount. Cash flows are associated with owning an instrument: stocks have dividends, bonds have coupons, futures have margin adjustments.

A *trading strategy* is a finite collection of strictly increasing stopping times, , and trades, indicating the number of shares to trade in each instrument. Trades accumulate to a *position*, where when .

The *value* of a position at time is : also called *marked-to-market*, is how much you would get from liquidating your position and the trades just executed assuming you could do that. The *amount* generated by the trading strategy at time is : you receive the cash flows associated with your existing position and pay for the trades you just executed.

A process is a martingale if . This is defined by where . If is understood we write this as . The usual notation is

A model is *arbitrage-free* if there is no trading strategy with , and for . The Fundamental Theorem of Asset Pricing states this is the case if and only if there exists a positive adapted process, , with

Note that if for all this says is a martingale.

A simple calculation using the definition of value and amount shows

For a strategy as above, . Since , , and we have , where the 0 subscript denote time .

Every model of the form where is a martingale and is a positive adapted process is arbitrage-free. This is immediate by substituting in the first equation.